

Econ 301: Managerial Economics

Topic 7: The Production Process & Costs

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Optimizing Production

While we have spent the last 2 topics looking at the consumer side, the lessons and techniques to optimization are very similar for the Firm.

- Focussing on the margin rather than average
- Optimization when $MB=MC$
- Optimization when the slope of ICs= The Slope of the budget line.

The Production Function

The production function is a mathematical function that defines the maximum amount of **output** that can be produced with a given set of inputs.

$$Q = F(K, L)$$

- Q is the level of output.
- K is the quantity of capital input.
- L is the quantity of labor input.

Short-Run versus Long-Run Decisions: Fixed and Variable Inputs

Short-run

- Period of time during which some factors of production (inputs) are ***fixed and*** constrain a manager's decisions
 - Usually Capital

Long-run

- Period of time during which all factors of production (inputs) are ***variable*** and can be adjusted by a manager

Measures of Productivity

Total product (*TP*)

- Maximum level of output that can be produced with a given amount of inputs

Average product (*AP*)

- A measure of the output produced per unit of input
- Average product of labor: $AP_L = \frac{Q}{L}$.
- Average product of capital: $AP_K = \frac{Q}{K}$.

Marginal product (*MP*)

The change in total product (output) attributable to the last unit of an input

- **Marginal product of labor:** $MP_L = \frac{\Delta Q}{\Delta L}$.
- **Marginal product of capital:** $MP_K = \frac{\Delta Q}{\Delta K}$.

Measures of Productivity in Action

- Consider the following production function when 5 units of labor and 10 units of capital are combined to produce: $Q = F(10, 5) = 150$.

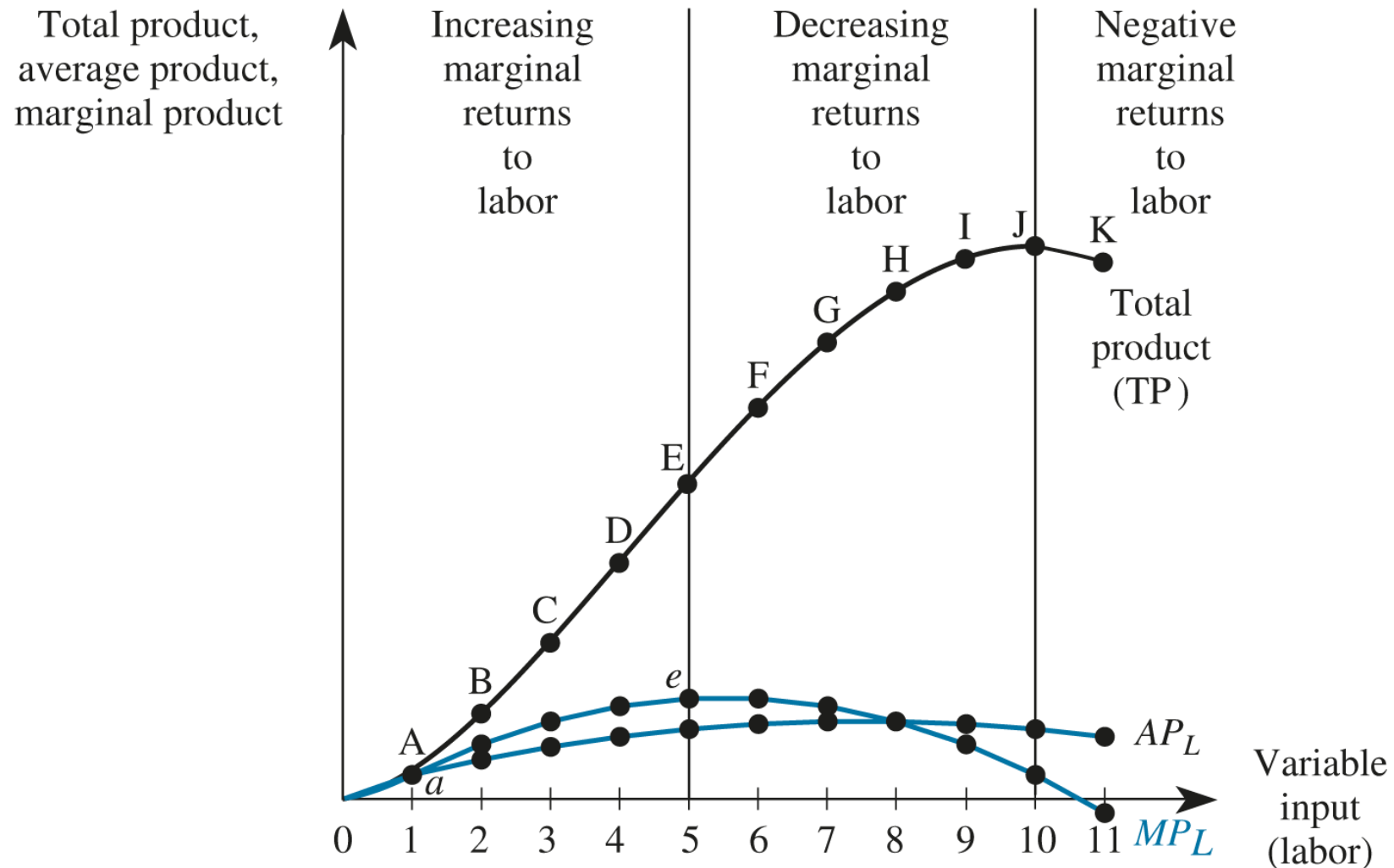
- Compute the average product of labor.

$$AP_L = \frac{150}{5} = 30 \text{ units per worker}$$

- Compute the average product of capital.

$$AP_K = \frac{150}{10} = 15 \text{ units capital unit}$$

Increasing, Decreasing, and Negative Marginal Returns



Increasing, Decreasing, and Negative Marginal Returns

Table 5–1 The Production Function

(1)	(2)	(3)	(4)	(5)	(6)
K^*	L	ΔL	Q	$\frac{\Delta Q}{\Delta L} = MP_L$	$\frac{Q}{L} = AP_L$
Fixed Input (Capital)	Variable Input (Labor)	Change in Labor	Output	Marginal Product of Labor	Average Product of Labor
[Given]	[Given]	$[\Delta(2)]$	[Given]	$[\Delta(4)/\Delta(2)]$	$[(4)/(2)]$
2	0	—	0	—	—
2	1	1	76	76	76
2	2	1	248	172	124
2	3	1	492	244	164
2	4	1	784	292	196
2	5	1	1,100	316	220
2	6	1	1,416	316	236
2	7	1	1,708	292	244
2	8	1	1,952	244	244
2	9	1	2,124	172	236
2	10	1	2,200	76	220
2	11	1	2,156	-44	196

The Role of the Manager in the Production Process

Produce output on the production function.

- Aligning incentives to induce maximum worker effort
 - More difficult than it seems! (Principal- Agent problem)

Use the right mix of inputs to maximize profits.

- To maximize profits when labor or capital vary in the short run, the manager will hire:
- Labor until the value of the marginal product of labor equals the wage rate:

$$VMP_L = w, \text{ where } VMP_L = P \times MP_L$$

- Capital until the value of the marginal product of capital equals the rental rate:

$$VMP_K = r, \text{ where } VMP_K = P \times MP_K$$

- Keep producing if an extra worker, or unit of capital brings in more revenue than the cost.

The Role of the Manager in the Production Process

- **Value marginal product:** The value of the output produced by the last unit of an input
- **Law of diminishing returns:** The marginal product of an additional unit of output will at some point be lower than the marginal product of the previous unit
- **Profit-Maximization input usage**
 - To maximize profits, use input levels at which *marginal benefit equals marginal cost*.
 - When the cost of each additional unit of labor is w , *the manager should continue to employ labor up to the point where $VMP_L = w$ in the range of **diminishing marginal product**.*

The Role of the Manager in the Production Process

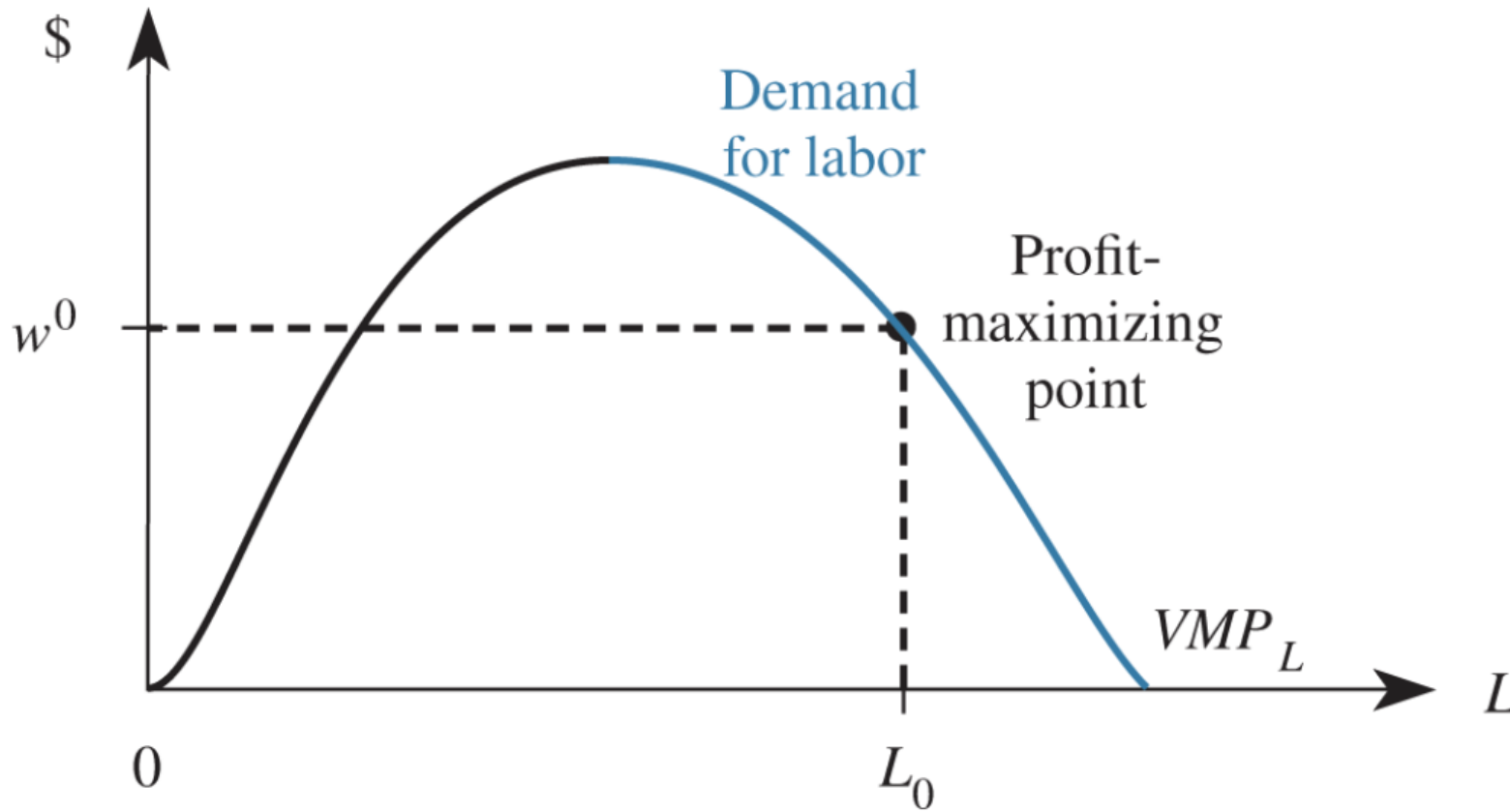


Figure 5–2
The Demand for Labor

Algebraic Forms of Production Functions

Commonly used algebraic production function forms:

- **Linear:** Assumes a perfect linear relationship between all inputs and total output

$$Q = F(K, L) = aK + bL, \text{ where } a \text{ and } b \text{ are constants}$$

- **Leontief:** Assumes that inputs are used in fixed proportions

$$Q = F(K, L) = \min\{aK, bL\}, \text{ where } a \text{ and } b \text{ are constants}$$

- **Cobb-Douglas:** Assumes some degree of substitutability among inputs

$$Q = F(K, L) = K^a L^b, \text{ where } a \text{ and } b \text{ are constants and usually } a+b=1$$

The Linear Production Function

Suppose that a firm's estimated production function is:

$$Q = 3K + 6L$$

How much output is produced when 3 units of capital and 7 units of labor are employed?

$$Q = F(3, 7) = 3(3) + 6(7) = 51 \text{ units}$$

The Leontief Production Function

Suppose that a firm's estimated production function is:

$$Q = \min\{3K, 6L\}$$

How much output is produced when 3 units of capital and 7 units of labor are employed?

$$Q = F(3,7) = \min\{3*3, 6*7\} = 9 \text{ units}$$

The Cobb-Douglas Production Function

Suppose that a firm's estimated production function is:

$$Q = K^{0.7}L^{0.3}$$

How much output is produced when 3 units of capital and 7 units of labor are employed?

$$Q = F(3,7) = 3^{0.7}7^{0.3} = 3.87 = 3 \text{ units}$$

Algebraic Measures of Productivity

Given the commonly used algebraic production function forms, we can compute the measures of productivity as follows:

- *Linear:*
 - Marginal products: $MP_K = a$ and $MP_L = b$.
 - Average products: $AP_K = \frac{aK + bL}{K}$ and $AP_L = \frac{aK + bL}{L}$.
- *Cobb-Douglas:*
 - Marginal products: $MP_K = aK^{a-1}L^b$ and $MP_L = bK^aL^{b-1}$.
 - Average products: $AP_K = \frac{K^aL^b}{K}$ and $AP_L = \frac{K^aL^b}{L}$.

Algebraic Measures of Productivity in Action

- Suppose that a firm produces output according to the production function.

$$Q = F(1, L) = (1)^{1/4} L^{3/4}$$

Which is the fixed input?

- Capital is the fixed input.

What is the marginal product of labor when 16 units of labor is hired?

$$MP_L = 1 \times \frac{3}{4} L^{-\frac{1}{4}} = 1 \times \frac{3}{4} (16)^{-\frac{1}{4}} = \frac{3}{8}$$

Isoquants and Marginal Rate of Technical Substitution

Isoquants capture the tradeoff between combinations of inputs that yield the same output in the long run, when all inputs are variable.

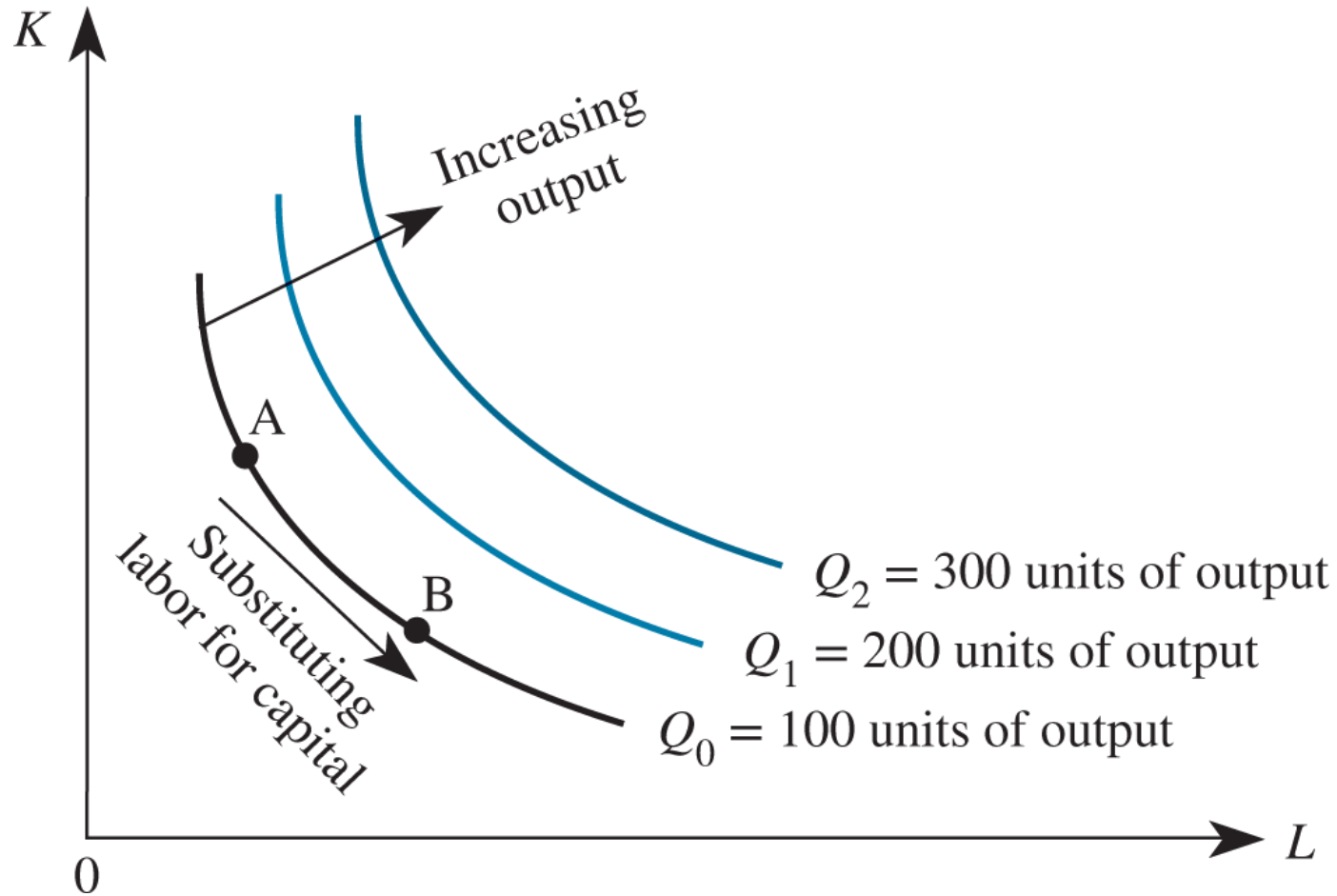
- The Indifference curves of production

Marginal rate of technical substitutions (***MRTS***)

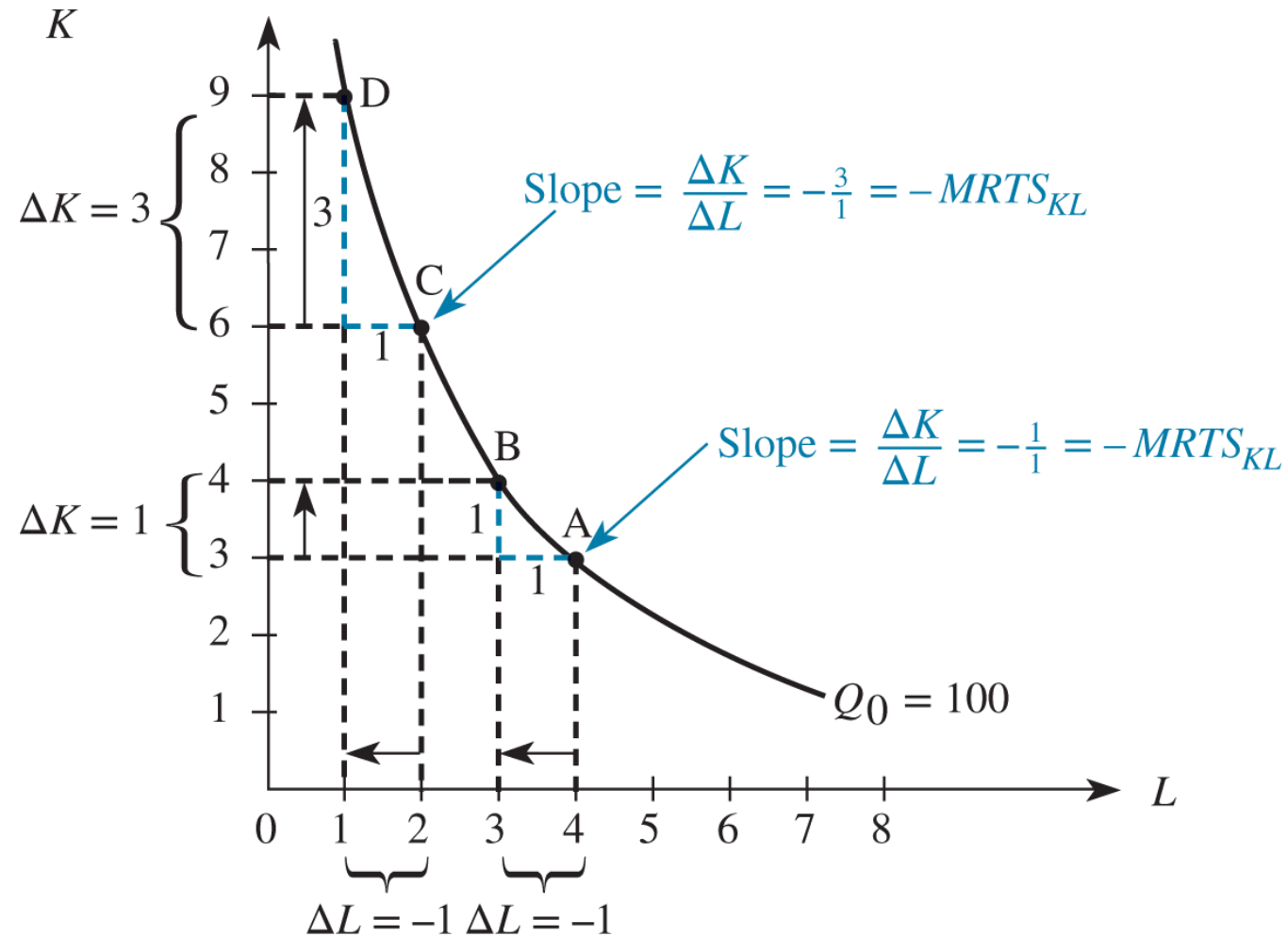
- The rate at which a producer can substitute between two inputs and maintain the same level of output
- Absolute value of the slope of the isoquant

$$MRTS_{KL} = \frac{MP_L}{MP_K}$$

A Family of Isoquants



The Marginal Rate of Technical Substitution



Isoquants for Linear and Leontief Functions

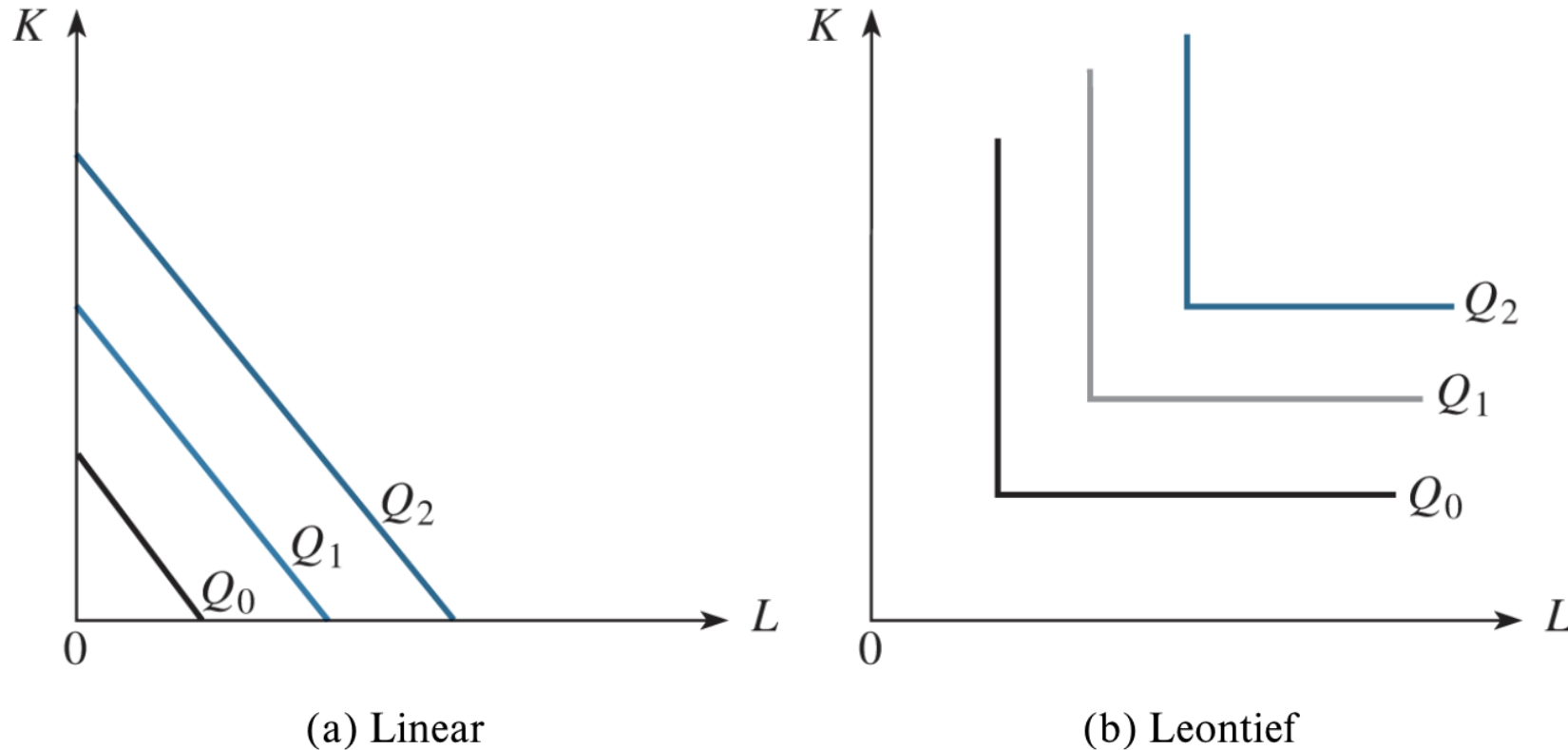


Figure 5–4
Linear and Leontief Isoquants

Isocost and Changes in Isocost Lines

Isocost (This is the budget line for Producers)

- Combination of inputs that yield the same cost:

$$wL + rK = C$$

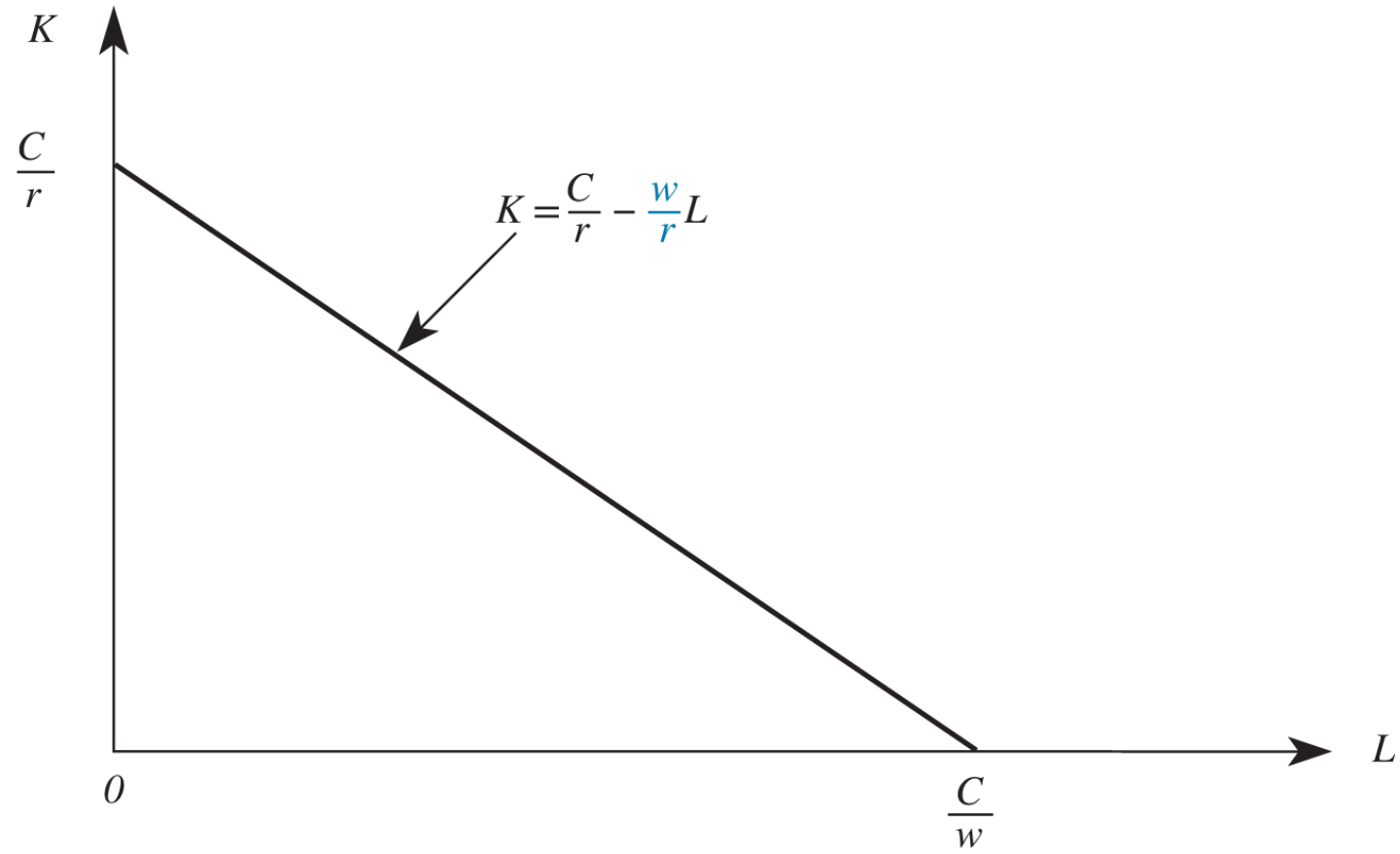
or re-arranging to the intercept-slope formulation:

$$K = \frac{C}{r} - \frac{w}{r}L$$

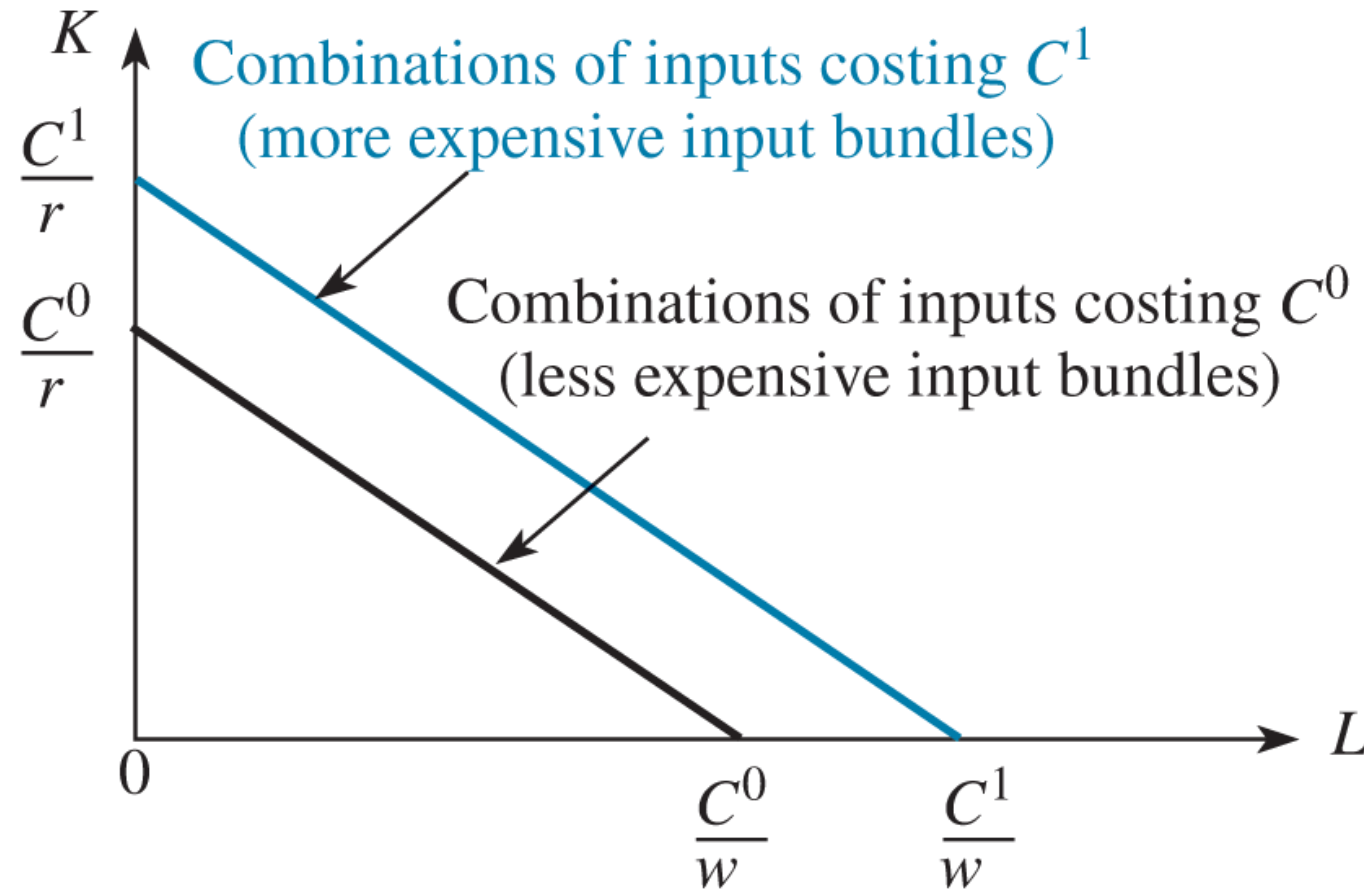
Changes in isocosts

- For given input prices, isocosts farther from the origin are associated with higher costs.
- Changes in input prices change the slopes of isocost lines.

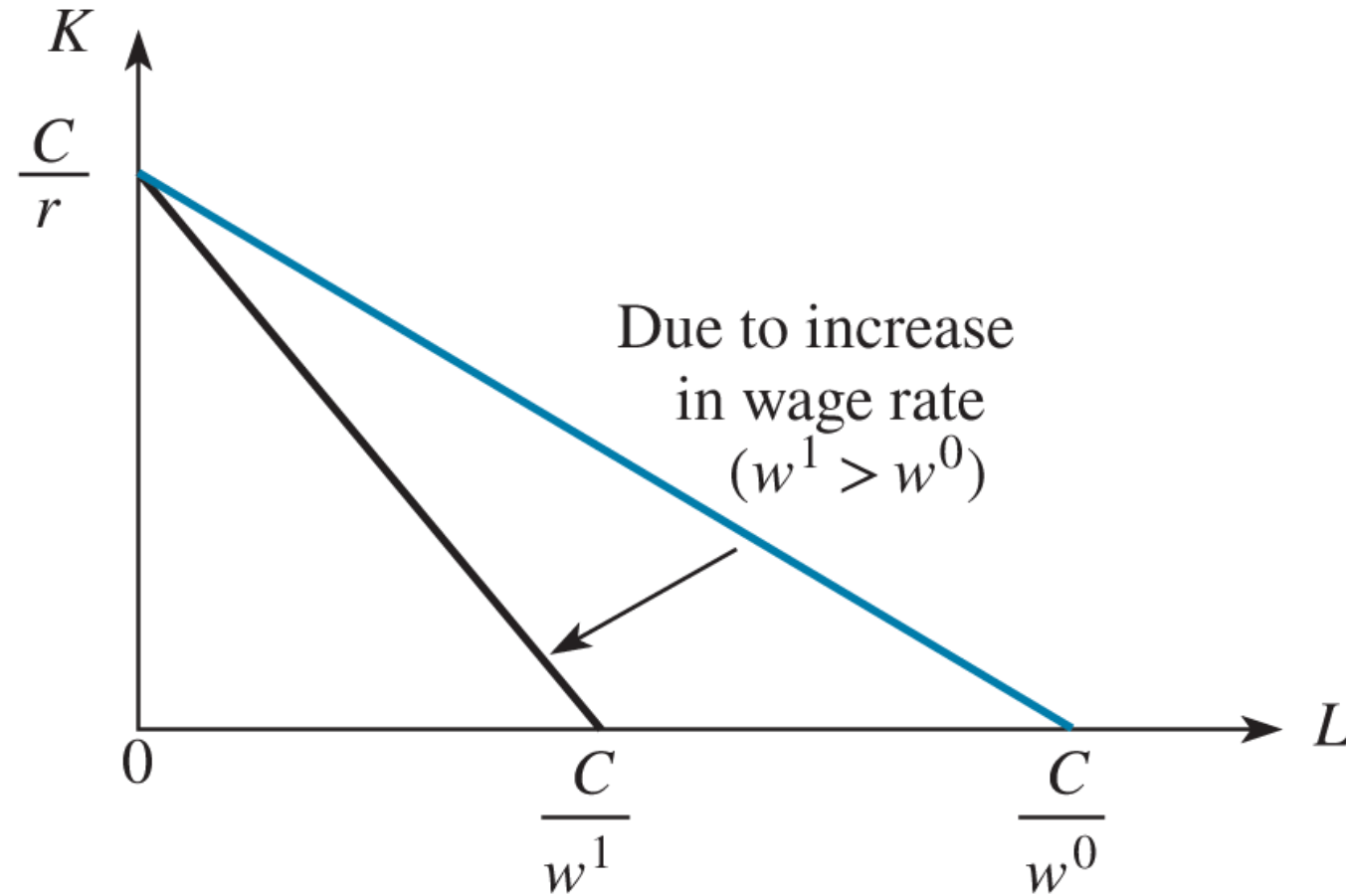
Isocosts



Changes in the Isocosts (Gotta spend money to make money)



Changes in the Isocost Line:



Cost Minimization and the Cost-Minimizing Input Rule

Cost minimization

- Producing at the **lowest possible cost**

Cost-minimizing input rule

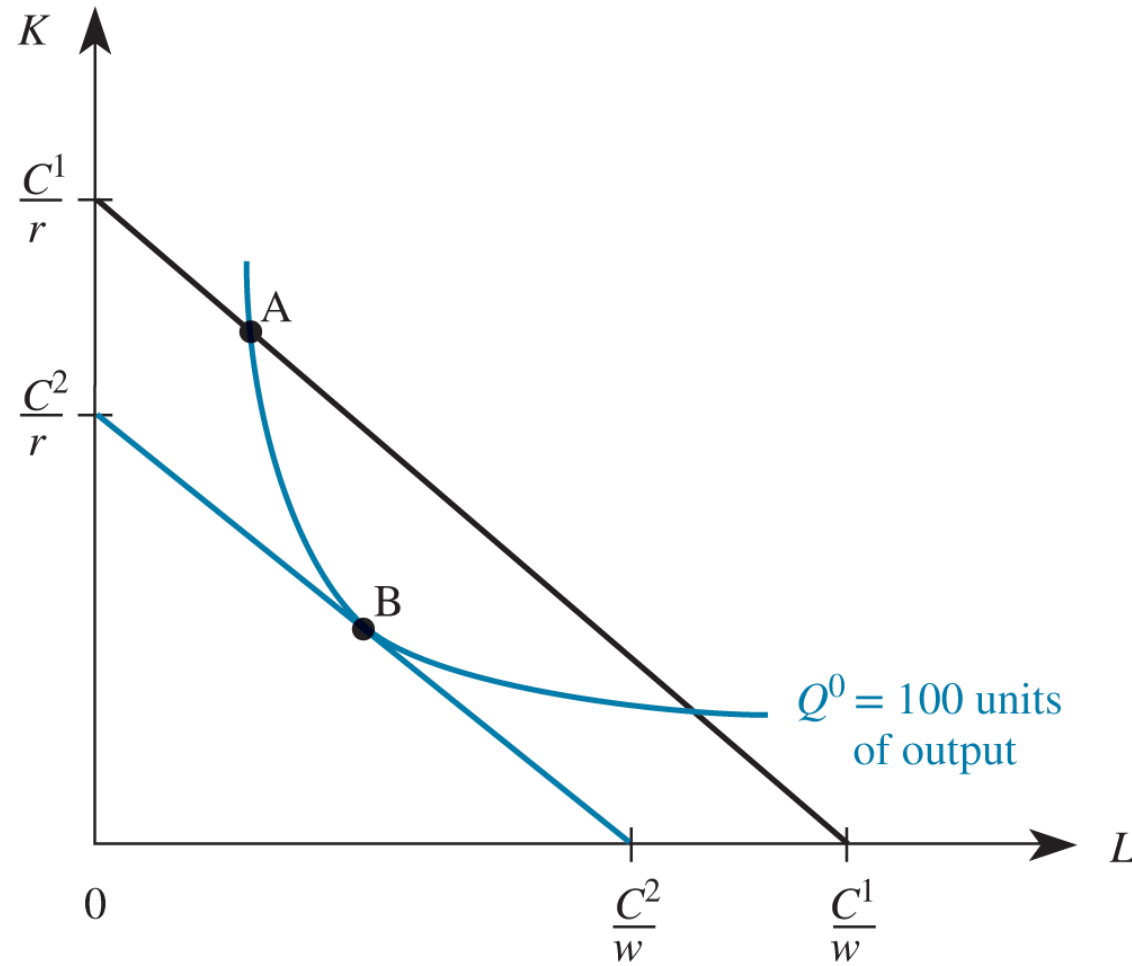
- Produce at a given level of output where the *marginal product per dollar spent is equal for all inputs*:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- Equivalently, a firm should employ inputs such that the *marginal rate of technical substitution equals the ratio of input prices*:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

Input Mix B Minimizes the Cost of Producing 100 units of Output



Example

Terry's Lawn Service rents five small push mowers and two large riding mowers to cut the lawns of neighborhood households. The marginal product of a small push mower is three lawns per day, and the marginal product of a large riding mower is six lawns per day. The rental price of a small push mower is \$10 per day, whereas the rental price of a large riding mower is \$25 per day. Is Terry's Lawn Service utilizing small push mowers and large riding mowers in a cost-minimizing manner?

Example

Let MP_S be the marginal product of a small push mower and MP_L be the marginal product of a large riding mower. If we let P_S and P_L be the rental prices of a small push mower and large riding mower, respectively, cost minimization requires that

$$\frac{MP_S}{P_S} = \frac{MP_L}{P_L}$$

Substituting in the appropriate values, we see that

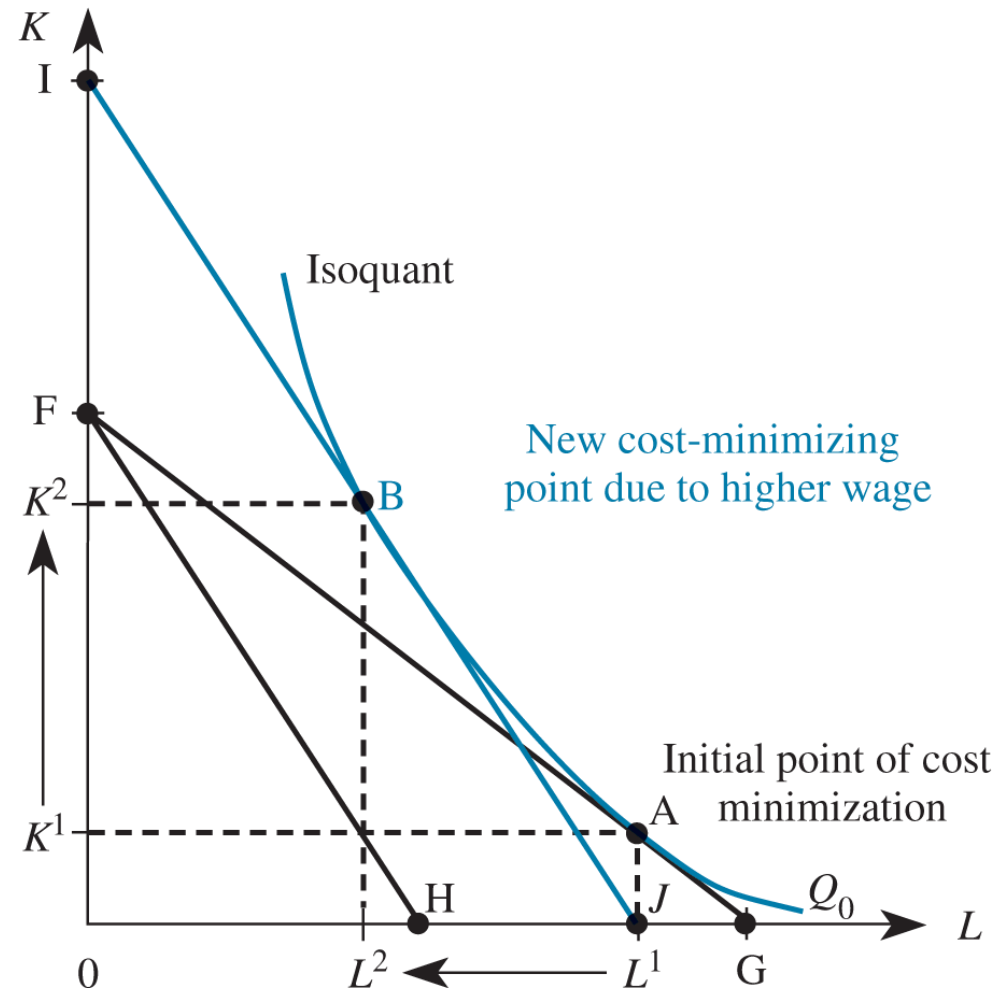
$$\frac{3}{10} = \frac{MP_S}{P_S} > \frac{MP_L}{P_L} = \frac{6}{25}$$

Thus, the marginal product per dollar spent on small push mowers exceeds the marginal product per dollar spent on large riding mowers. Riding mowers are twice as productive as push mowers, but more than twice as expensive. The firm clearly is not minimizing costs and thus should use fewer riding mowers and more push mowers.

Optimal Input Substitution

- To minimize the cost of producing a given level of output, the firm should use less of an input and more of other inputs when that input's price rises.
- Example of Cost minimization: [The Founder](#)

Substituting Capital for Labor Due to Increase in the Wage Rate

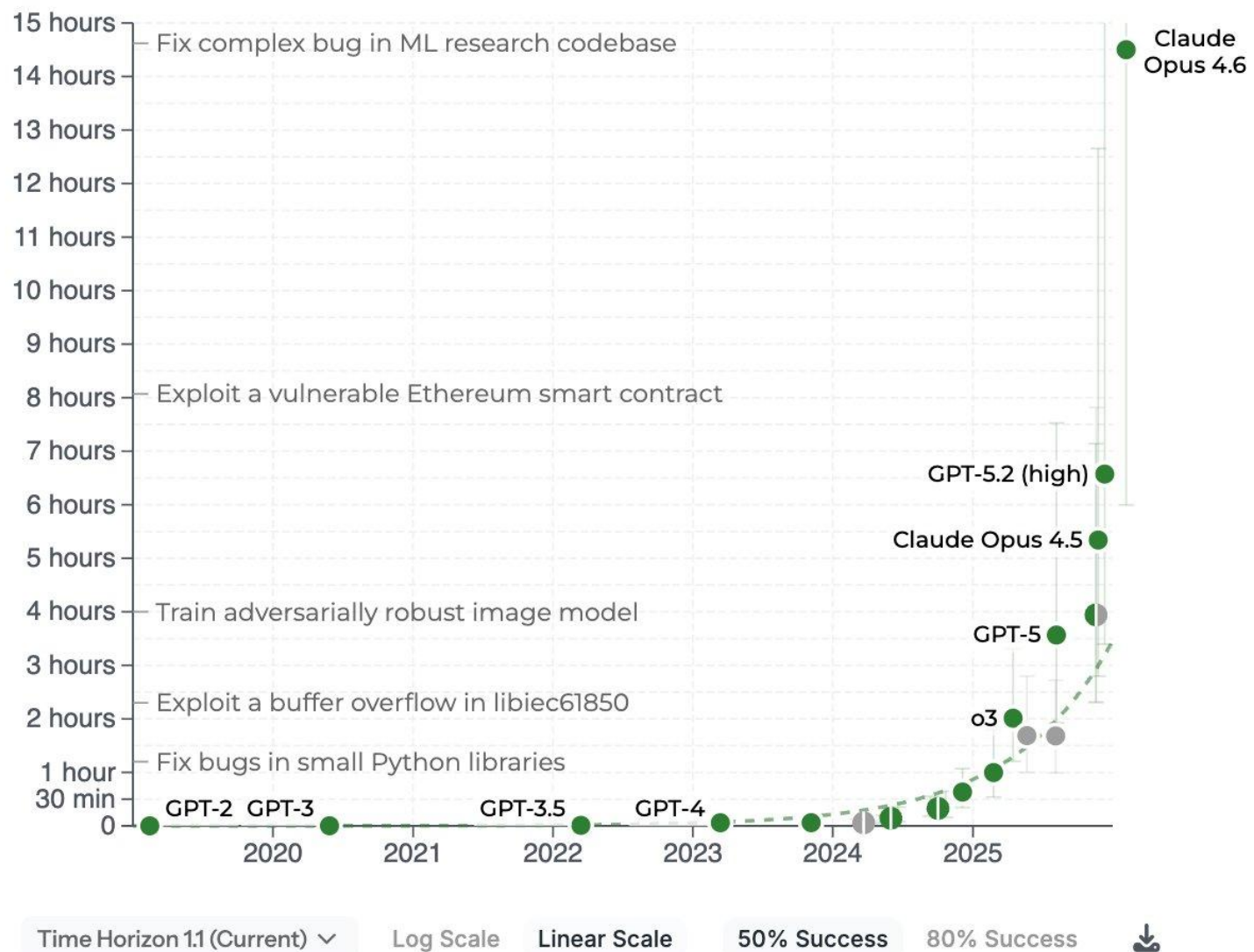


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AI & Cost Minimization

- Does AI augment or replace Capital/Labor?
- I don't think it's a hot take to suggest it's going to have large effects on firms and the labor market in the next couple of years (in fact, we are already seeing it)!
- What work will exist in the post-AGI economy?
 - What skills are necessary?

METR Evals



The Cost Function

Mathematical relationship that relates cost to the cost-minimizing output associated with an isoquant

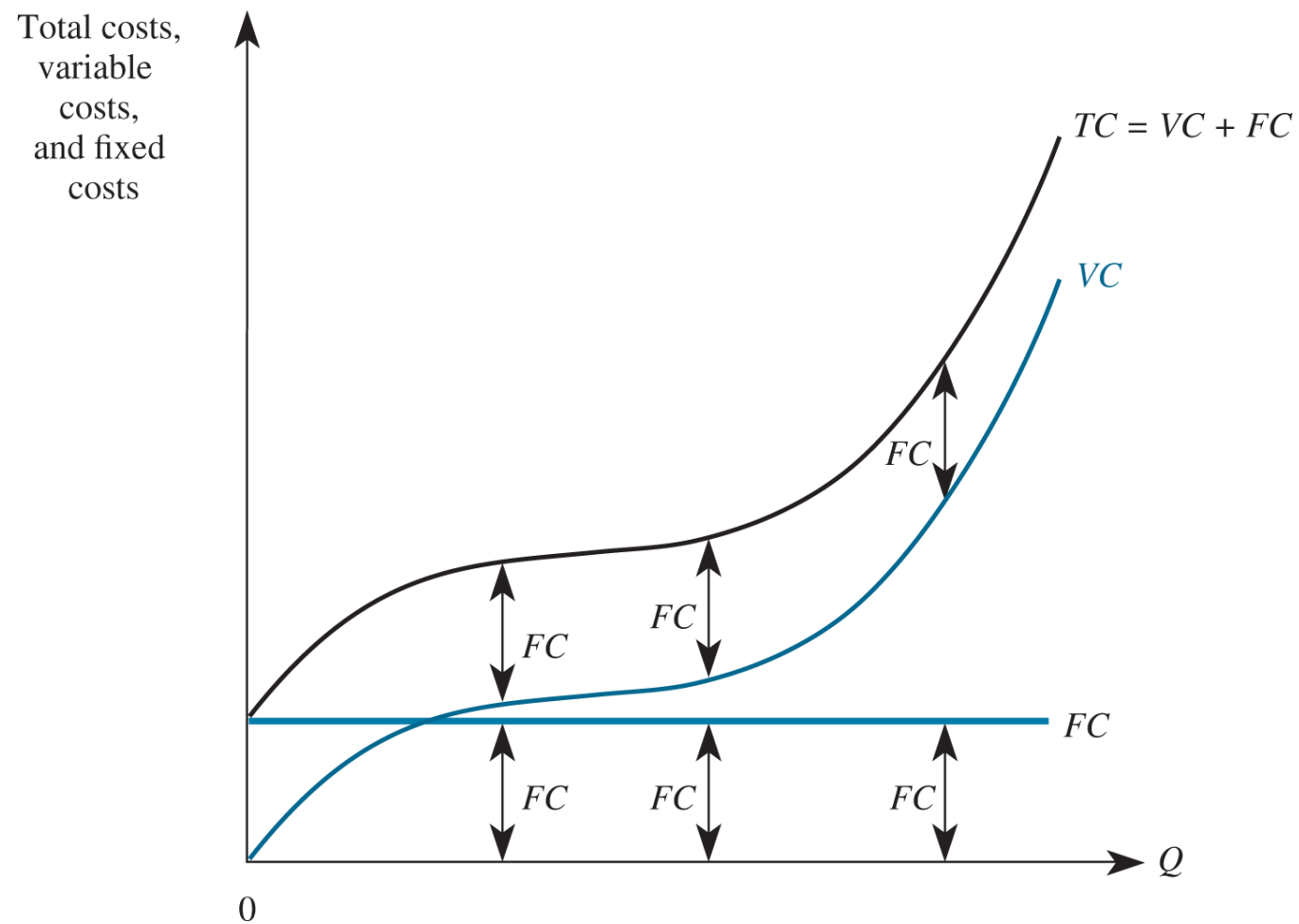
Short-run costs.

- **Fixed costs (FC):** do not change with changes in output; include the costs of fixed inputs used in production
- **Variable costs [$VC(Q)$]:** • costs that change with changes in outputs; include the costs of inputs that vary with output
- **Total costs:** $TC(Q) = FC + VC(Q)$

Long-run costs.

- All costs variable
- No fixed costs

The Relationship among Costs



Average and Marginal Costs

Average costs.

- Average fixed cost: $AFC = \frac{FC}{Q}$.
- Average variable costs: $AVC = \frac{VC(Q)}{Q}$.
- Average total cost: $ATC = \frac{C(Q)}{Q}$.

Marginal cost (MC).

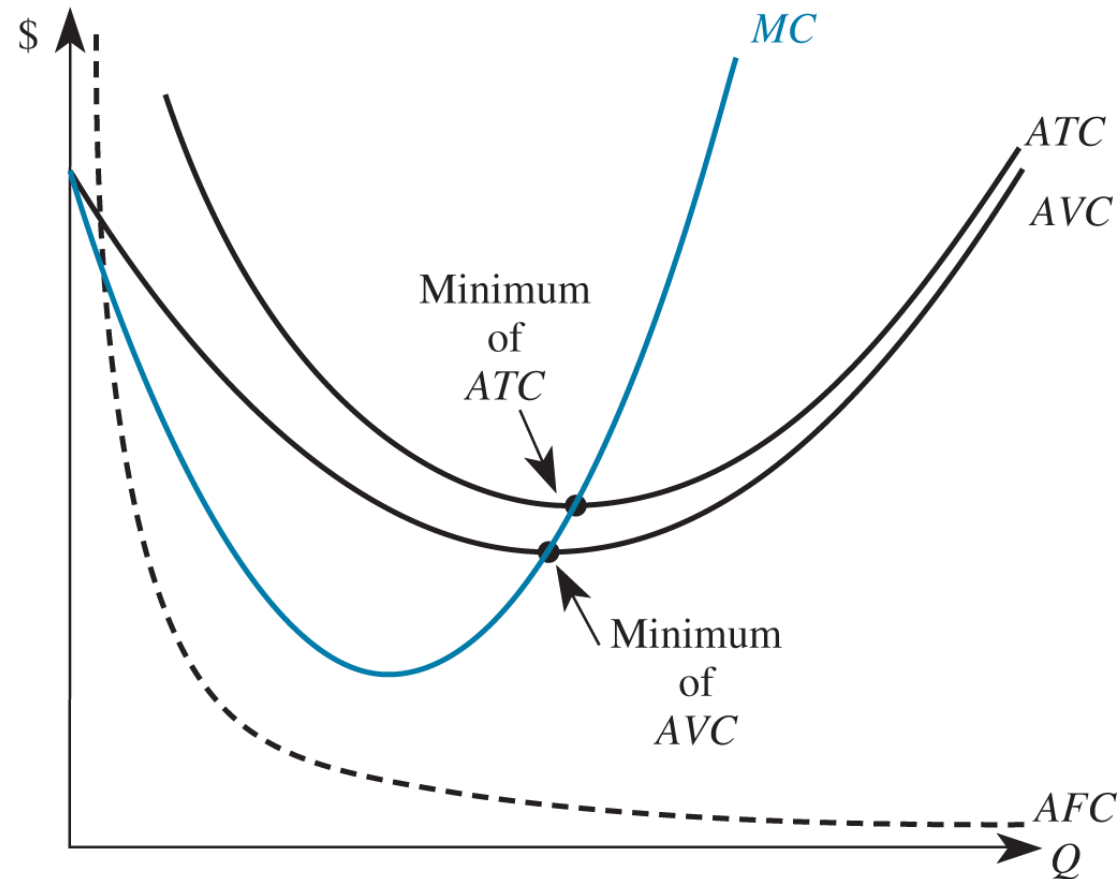
- The (incremental) cost of producing an additional unit of output
- $MC = \frac{\Delta C}{\Delta Q}$.

The Relationship among Costs

Table 5–5 Derivation of Average Costs

(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>Q</i>	<i>FC</i>	<i>VC</i>	<i>TC</i>	<i>AFC</i>	<i>AVC</i>	<i>ATC</i>
Output	Fixed Cost	Variable Cost	Total Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost
[Given]	[Given]	[Given]	$[(2) + (3)]$	$[(2)/(1)]$	$[(3)/(1)]$	$[(4)/(1)]$
0	\$2,000	\$ 0	\$2,000	—	—	—
76	2,000	400	2,400	\$26.32	\$5.26	\$31.58
248	2,000	800	2,800	8.06	3.23	11.29
492	2,000	1,200	3,200	4.07	2.44	6.50
784	2,000	1,600	3,600	2.55	2.04	4.59
1,100	2,000	2,000	4,000	1.82	1.82	3.64
1,416	2,000	2,400	4,400	1.41	1.69	3.11
1,708	2,000	2,800	4,800	1.17	1.64	2.81
1,952	2,000	3,200	5,200	1.02	1.64	2.66
2,124	2,000	3,600	5,600	0.94	1.69	2.64
2,200	2,000	4,000	6,000	0.91	1.82	2.73

The Relationship between Average and Marginal Costs



*The *MC* curve intersects the *ATC* and *AVC* curves at their minimum points.

- [Access the text alternative for slide images.](#)

Fixed and Sunk Costs

Fixed costs

- Cost that does not change with output

Sunk cost

- Cost that is forever lost after it has been paid

Irrelevance of Sunk Costs

- A decision maker should ignore sunk costs to maximize profits or minimize losses.

Algebraic Forms of Cost Functions

The cubic cost function: costs are a cubic function of output; provides a reasonable approximation to virtually any cost function.

$$C(Q) = f + aQ + bQ^2 + cQ^3$$

Where a , b , c , and f are constants; f represents fixed costs.

Marginal cost function is:

$$MC(Q) = a + 2bQ + 3cQ^2$$

Managers can use regression analysis to estimate these parameters (see Table05-07.xls).

Table 5-7

Regression Estimates for a Cubic Cost Function

	Coefficients	Standard Error	t-Stat	P-value	Lower 95%	Upper 95%
Intercept	1034.419	47.578	21.742	8.6E-49	940.430	1128.409
Quantity	21.670	8.921	2.429	0.016	4.047	39.293
Quantity ²	1.093	0.532	2.055	0.042	0.043	2.144
Quantity ³	0.087	0.010	8.571	1.0E-14	0.067	0.107

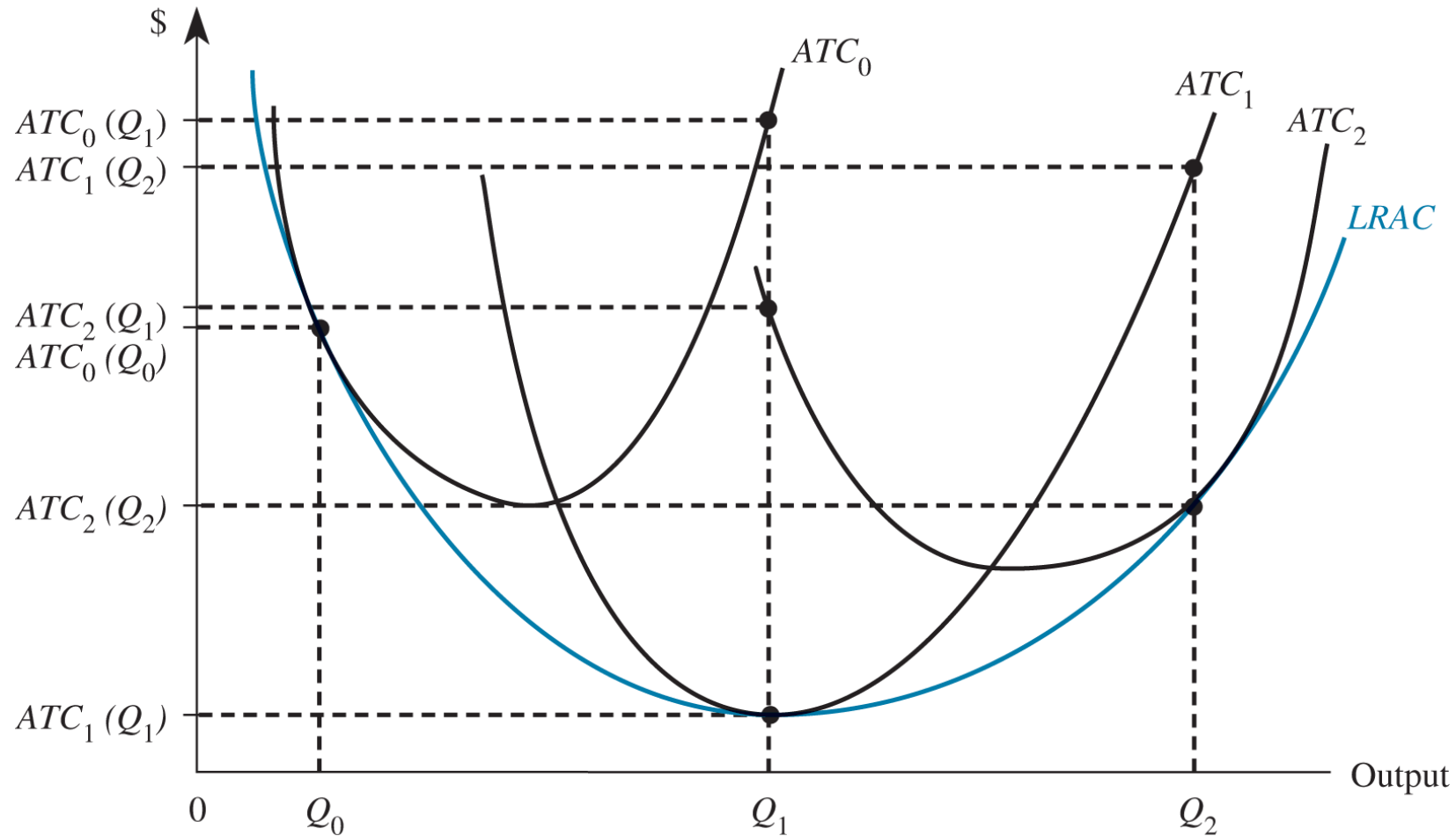
Long-Run Costs

In the long run, all costs are variable since a manager is free to adjust levels of all inputs.

Long-run average cost curve

- A curve that defines the minimum average cost of producing alternative levels of output allowing for optimal selection of both fixed and variable factors of production.

Optimal Plant Size and Long-Run Average Cost



Economies of Scale

Economies of scale

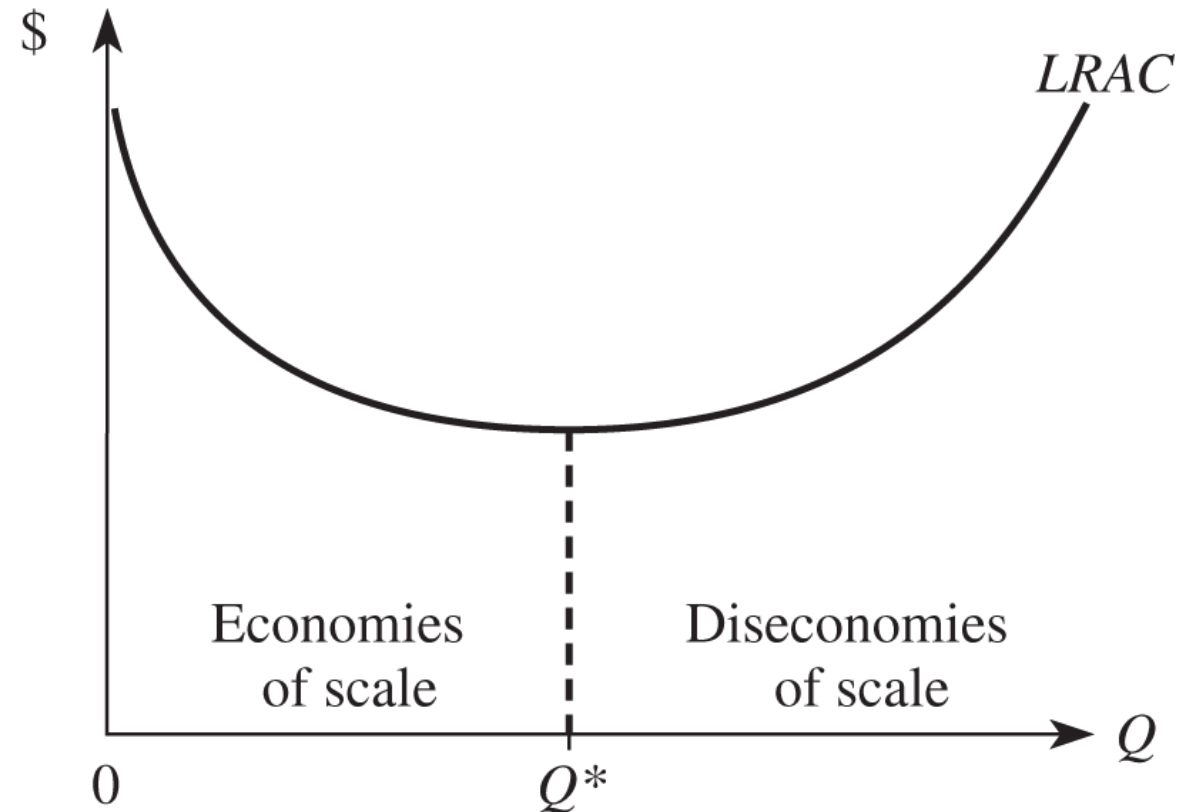
- Declining portion of the long-run average cost curve as output increases

Diseconomies of scale

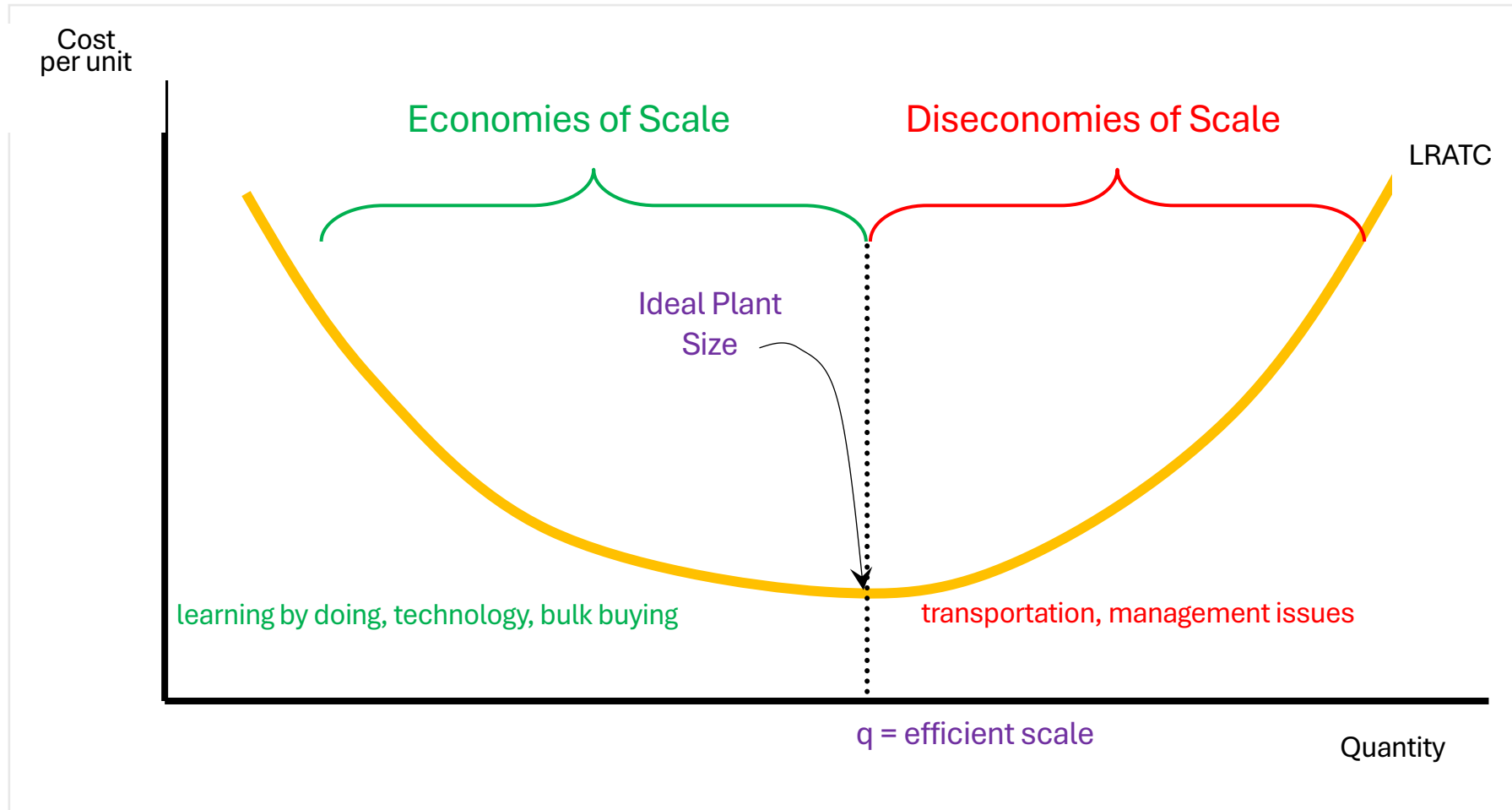
- Rising portion of the long-run average cost curve as output increases

Constant returns to scale

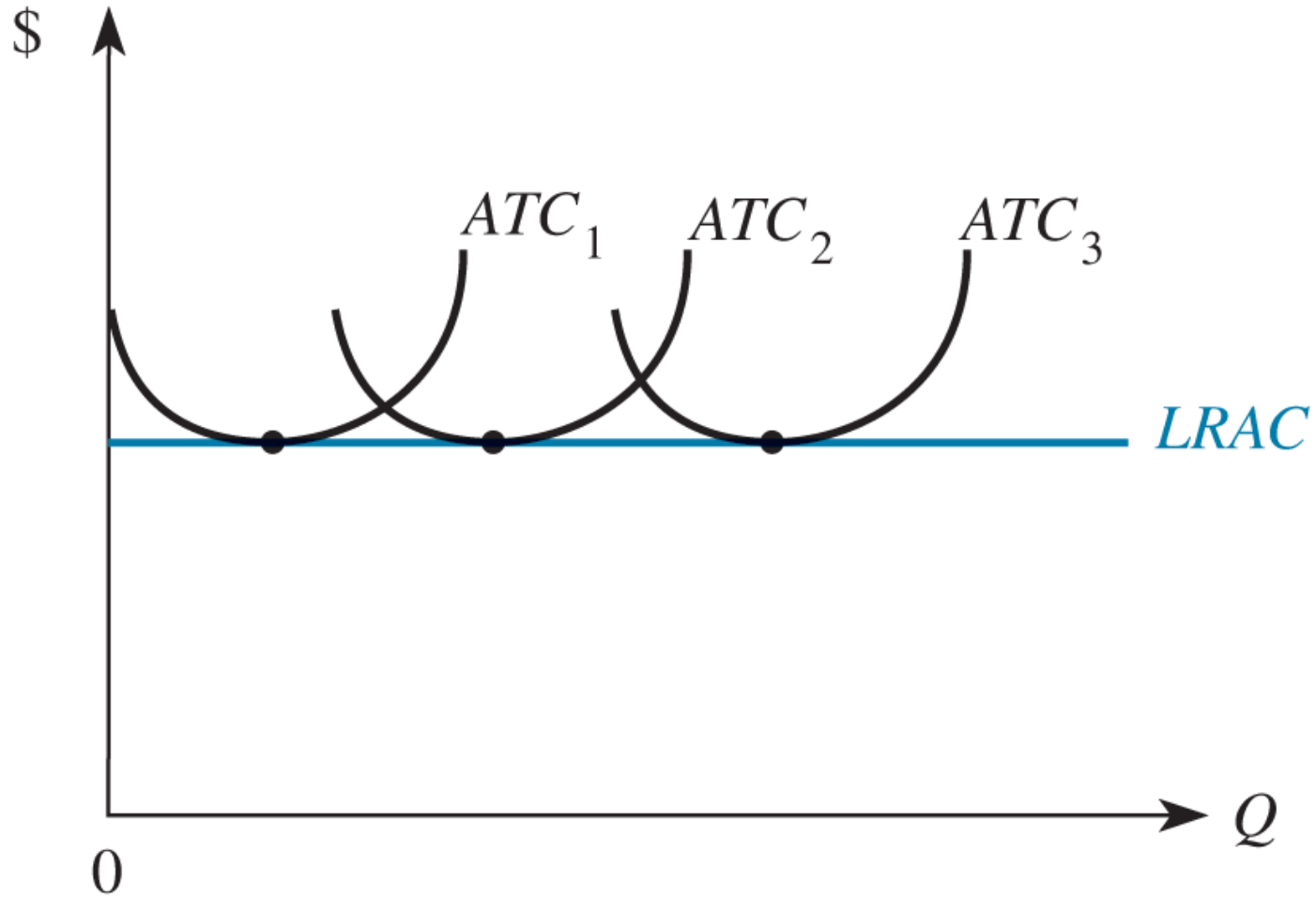
- Portion of the long-run average cost curve that remains constant as output increases



Economies of Scale



Constant Returns to Scale



Multiple-Output Cost Function

Economies of scope

- Exist when the total cost of producing Q_1 and Q_2 together is less than the total cost of producing each of the type of output separately

$$C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)$$

Cost complementarity

- Exist when the marginal cost of producing one type of output decreases when the output of another good is increased

$$\frac{\Delta MC_1(Q_1, Q_2)}{\Delta Q_2} < 0$$

Examples

Economies of Scope:

- If you produce both cars and trucks you will already have a lot of the infrastructure in place to produce the other one (similar machines, labor talent etc).

Cost Complementarity

- Doughnuts and doughnut holes. The firm can make these products separately or jointly. But the cost of making additional doughnut holes is lower when workers roll out the dough, punch the holes, and fry both the doughnuts and the holes instead of making the holes separately

Algebraic Form for a Multiproduct Cost Function

$$C(Q_1, Q_2) = f + aQ_1Q_2 + (Q_1)^2 + (Q_2)^2$$

For this cost function:

$$MC_1 = aQ_2 + 2Q_1$$

- When $a < 0$, an increase in Q_2 reduces the marginal cost of producing product 1.
- If $a < 0$, this cost function exhibits cost complementarity.
- If $a > 0$, there are no cost complementarities.
- Exhibits economies of scope whenever $f - aQ_1Q_2 > 0$.